Step Response Analysis

By: John Getty
Laboratory Director
Engineering Department
University of Denver
Denver, CO

Purpose:

Examine the behavior of under-damped, critically-damped and over-damped second-order circuits.

Equipment Required:

- 1 - Agilent 54622A Deep Memory Oscilloscope or Agilent 54600B Oscilloscope
- 1 - Agilent 33120A Function Generator
- 1 - Protoboard
- 1 - Multimeter
- 1 - RCL meter (can be shared)
- 1 - 10-kΩ Potentiometer
- 1 - 0.25-H Inductor
- 1 - 0.1-µF capacitor

Prelab:

Read Sections 8-5 , 8-6 and 8-7 in the text[*].

1. The characteristic equation

   A simplified model for an inductor, shown in Fig. 1, consists of an ideal inductor in series with a parasitic resistance R_P. For all prelab calculations, use the inductor model in Fig. 1 and assume $R_P = 100 \ \Omega$.
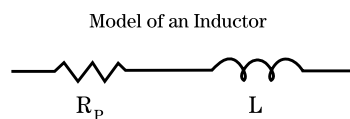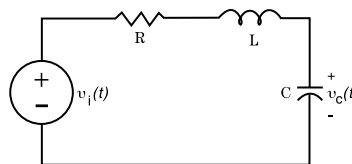
   Model of an Inductor

   R_P $\qquad$ L

   Figure 1

   Figure 2

   a. For R = 10 kΩ, L = 0.25 H, and C = 0.1 µF, find the roots of the characteristic equation of the circuit in Fig. 2.

   b. Find the critical resistance R_c that will result in two equal roots. Find an expression for $v_c(t)$ if $v_i(t) = 2u(t)$ V and $R = R_c$.

   c. For R = 150 Ω , find the roots of the characteristic equation of the circuit in Fig. 2.

2. CCA

The procedure section of this lab exercise examines all three cases for an RLC circuit, overdamped (Case A), critically damped (Case B) and underdamped (Case C). This CCA exercise predicts the circuit response in these three states.

Enter the circuit of Fig. 3 into a CCA program. In all cases, $v_i(t)$ is a square wave source with $V_{pp} = 2$ V. Use the transient analysis method. The output is the circuit voltage across the capacitor, $v_c(t)$. R_{TH} should be set to the output resistance of the function generator used in your laboratory. R_p is 100 Ω . Plot and print out the circuit response for two transitions of the source, one positive-going step and one negative-going step, for each "case" listed below.
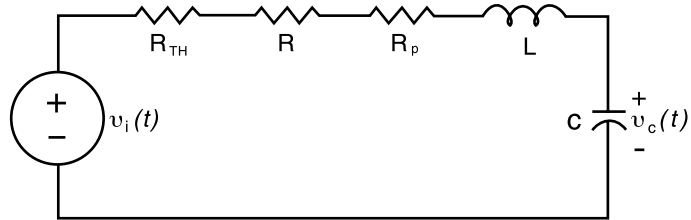

Figure 3

Case A: Set the value of R to 10 kΩ .

Case B: Compute the value of R required to critically damp the circuit of Fig. 3, from $R_C = R + R_P + R_{TH}$. The resistance R in the circuit of Fig. 2 represents the total circuit resistance, including R_P and R_{TH}. To achieve the critically damped state for the CCA — and in the laboratory — R will need to be determined by subtracting R_P and R_{TH} from R_C. Set R in the CCA circuit to the value needed to place the circuit in the critically damped state.

Case C: Set the value of R to 0 Ω.

For each run, compute ω_o and ζ, and write these parameters on the plot for that run.

Procedure:

1. Prepare the circuit

   a. Set the function generator to 100 Hz, square wave output, 2 V_{pp} and no DC offset. Use the oscilloscope to ensure that your settings are correct.

   b. Measure and record the parasitic resistance of the inductor. This can be accomplished by simply measuring the resistance of the inductor with the DMM.
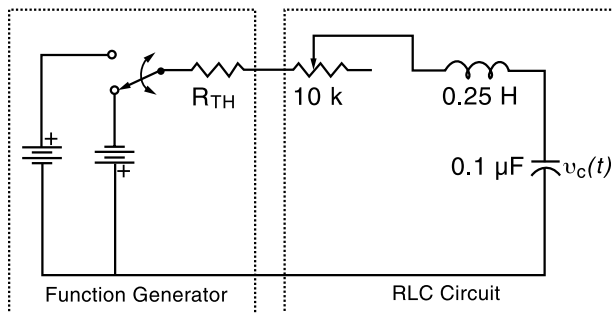

Function Generator   RLC Circuit

Figure 4

    c. The function generator model shown in Fig. 4 is a reminder that the square wave source can assume only two discrete voltages, and that the function generator has a finite output resistance. Draw a schematic similar to Fig. 4 in your lab journal, labeling the R_{TH} with the output resistance of the function generator. Construct the circuit of Fig. 4. Connect the oscilloscope across the capacitor and turn on the function generator. When wired as shown in Fig. 4, the 10-kΩ potentiometer is referred to as a variable resistor. Adjust the variable resistor over its entire range and note how the output waveform $v_c(t)$ changes.

2. Plot the circuit step response

For each of the following sections, set the total circuit resistance to the specified value and draw a sketch of the resulting output waveform. Make your sketches sufficiently accurate to measure the decay time constants (α) for Cases A, B and C, and the damped frequency (β) for Case C.

    a. Set the resistance of the variable resistor to its maximum, and measure its value. The total circuit resistance is computed by adding the actual value of the variable resistor, the function generator output resistance and the parasitic resistance of the inductor. Reconnect the variable resistor and turn on the function generator. In your journal, sketch $v_c(t)$ as displayed by the oscilloscope. This circuit is overdamped and meets the criteria for Case A as defined in the text.

    b. Remove the variable resistor and set its value so that the total circuit resistance will be equal to the critical resistance found in the prelab. Without changing its value, reconnect the variable resistor into the circuit, and sketch $v_c(t)$. This circuit is critically damped, and meets the criteria for Case B.

    c. Remove the variable resistor from the circuit and replace it with a jumper wire. The total circuit resistance is now the sum of the output resistance of the function generator and the parasitic resistance of the inductor. This circuit is underdamped, and meets the criteria for Case C.

3. Measure the response parameters

The "Exponential Waveforms" experiment describes a procedure for measuring the time constant, using Eq. 1.

$$T_C = \frac{t_2 - t_1}{\ln\left(\dfrac{v(t_1) - FC}{v(t_2) - FC}\right)}$$

$$(1)$$

Use this method to determine the time constant of the plots of $v_c(t)$ as described below.

In all three cases A, B and C, the waveform $v_c(t)$ is a sum of terms. Verifying that these plots represent the solution to the second-order linear differential equation requires an understanding of the equations that describe them. The roots of the characteristic equations of these circuits appear as an exponent of *e*. For this reason, the roots are related to the "time constant" for that term of the natural response by $T_C = 1/\alpha$.

The approach to verifying these solution waveforms is dependent on the case.

Case A: For Case A, the two time constants ($T_{C1} = 1/\alpha_1$ and $T_{C2} = 1/\alpha_2$ in Eq. 2) are significantly different. The natural response for Case A is of the form

$$y_N(t) = K_1 e^{-\alpha_1 t} + K_2 e^{-\alpha_2 t} \tag{2}$$

For the shorter of the two time constants, compute $5T_c$. Use this time as t_1 for Eq. 1. Collect the remaining values for Eq. 1 from the sketch of the overdamped circuit, and compute the time constant. This procedure permits one of the terms to decay to essentially zero.

Case B: This case has two equal roots, and therefore the time constants for the two terms are the same.

$$y_N(t) = K_1 e^{-\alpha t} + K_2 t e^{-\alpha t} \tag{3}$$

However, the second term in the natural response for Case B is multiplied by the ramp function, t. Use Eq. 1 to calculate α. However, start the measurement at $t_1 = 0$ (at the transition where the source changes from one value to the other) , and pick t_2 as close as possible to t_1. This will reduce the contribution made by the second term in the natural response equation for Case B.

Case C: The equation describing the natural response for Case C can be rearranged to show that the equation is the product of an exponential function with a time constant of $1/\alpha$ (the "envelope" shown in Fig. 5), times the sum of two sinusoids of the same frequency β, as shown in Eq. 4.

$$y_N(t) = e^{-\alpha t}(K_1 \cos\beta t + K_2 \sin\beta t) \tag{4}$$

The time constant is determined from the exponential envelope bounding the sinusoid, shown in Fig. 5. Note that the only points on $v_c(t)$ that intersect the envelope are just past the peaks (and valleys) in the response plot. Use two of these peaks for t_1 and t_2 in Eq 1. β is measured from the period T_o of the sinusoid $v_c(t)$.
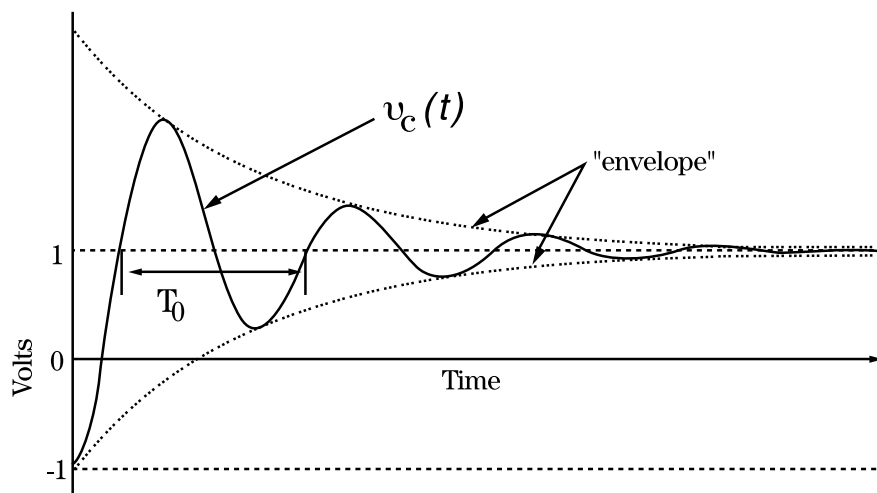

Figure 5

4. Summarize the results and compute error

Build an appropriate table and for each of the Cases A, B and C, compare the measured values of a and b with the computed values. Since the measured values represent a sum of terms, you will need to explain how you justify the comparisons. For each case, state if the circuit is over, under, or critically damped.

Conclusion

In your conclusion, answer the following questions:

a. How does decreasing R affect α, β, ω_o, ζ?

b. How does decreasing C affect α, β, ω_o, ζ?

c. What are the sources of error?

Note that the answers are, in some cases, dependent on the "case" of the circuit, that is whether the circuit is over, under or critically damped. Use the characteristic equation to determine the answers, and present the results in the form of a table similar to the one below. Some of the answers are provided for you.

Case	A	A	B	C	C	All	All
Variable	α_1	α_2	α	α	β	ω_o	ζ
R $\downarrow$		$\uparrow$					
C $\downarrow$			*	$\rightarrow$			

* Not applicable. (Good question! Why is it not applicable?)

* Roland E. Thomas and Albert J. Rosa, <u>The Analysis and Design of Linear Circuts</u>, Prentice Hall, (New Jersey, 1994)